Exercise 22

For the following exercises, consider an athlete running a 40-m dash. The position of the athlete is given by $d(t) = \frac{t^3}{6} + 4t$, where d is the position in meters and t is the time elapsed, measured in seconds.

Compute the average velocity of the runner over the given time intervals.

a. $[1.95, 2.05]$

b. $[1.995, 2.005]$

c. $[1.9995, 2.0005]$

d. $[2, 2.00001]$

Solution

The average velocity is calculated by

$$v_{\text{avg}} = \frac{d(t_2) - d(t_1)}{t_2 - t_1}.$$

Over the interval $[1.95, 2.05]$ the average velocity is

$$v_{\text{avg}} = \frac{d(2.05) - d(1.95)}{2.05 - 1.95} = \frac{\left[\frac{(2.05)^3}{6} + 4(2.05)\right] - \left[\frac{(1.95)^3}{6} + 4(1.95)\right]}{2.05 - 1.95} \approx 6.0004167.$$

Over the interval $[1.995, 2.005]$ the average velocity is

$$v_{\text{avg}} = \frac{d(2.005) - d(1.995)}{2.005 - 1.995} = \frac{\left[\frac{(2.005)^3}{6} + 4(2.005)\right] - \left[\frac{(1.995)^3}{6} + 4(1.995)\right]}{2.005 - 1.995} \approx 6.000004167.$$

Over the interval $[1.9995, 2.0005]$ the average velocity is

$$v_{\text{avg}} = \frac{d(2.0005) - d(1.9995)}{2.0005 - 1.9995} = \frac{\left[\frac{(2.0005)^3}{6} + 4(2.0005)\right] - \left[\frac{(1.9995)^3}{6} + 4(1.9995)\right]}{2.0005 - 1.9995} \approx 6.00000004167.$$

Over the interval $[2, 2.00001]$ the average velocity is

$$v_{\text{avg}} = \frac{d(2.00001) - d(2)}{2.00001 - 2} = \frac{\left[\frac{(2.00001)^3}{6} + 4(2.00001)\right] - \left[\frac{(2)^3}{6} + 4(2)\right]}{2.00001 - 2} \approx 6.00001.$$